

Geometry and modeling exercises using the observation of Venus occultation by the Moon, September 18, 2017

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Abstract

The result of the only known good quality observations of September 18, 2017 Venus occultation by the Moon is presented in this article. This kind of observation can be done by students using a common camera, and the analysis can be designed as a mathematics and physics exercise. Several observations were carried out in Malang and Jakarta, but high quality data were obtained only in Jakarta. An analysis of the occultation geometry then showed that the shortest distance between Venus and the center of the Moon disk was 249 arcsec. We were also able to fit the occultation light curve with a simple geometric model using an additional correction factor $(1 + \sin^2 \phi)$ to compensate for the hill-shaped intensity variation in the Venus disk.

Keywords: planet occultation, Venus, Moon

(Some figures may appear in colour only in the online journal)

1. Introduction

Astronomical phenomena are usually interesting for people to watch. Many astronomical phenomena such as cosmic microwave background (CMB) radiation from the Universe (e.g., [1]) are so interesting that they fascinate students and attract them to study science. The occultation of bright planets by the Moon is particularly interesting. It is easily observed by the naked eye, and it fascinates people to see the planet disappear behind the Moon and later reappear. There

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are many reasons why occultation also makes for an interesting and attractive real research exercise for undergraduate students to record phenomena. First, the observation can be carried out with a common camera, with or without a small telescope. Although such occultation no longer has a high scientific value, it still has certain originality because each occultation is unique, and the data extracted may be useful in the future, for example, in research on the evolution of Earth's rotation [2] or research on planetary atmosphere [3], etc. The importance of occultation observations was also described in [4], and for the occultation of planets and asteroids by the Moon, rich resources can be found in [5].

An occultation is also a good study subject for student geometry, image processing and modeling exercises. Learning mathematics through a case study is more attractive than the usual problem solving, especially during the COVID-19 pandemic, when assessing student's competence by exam has proven to be more difficult than assessing students through this kind of project. Teachers would be able to ask students to work independently from their home using common camera and consult about their work online.

The occultation of Venus by the Moon on September 18, 2017 is one example of a project that teachers could provide for their students. It was clearly observed from Australia, New Zealand, Indonesia and Papua New Guinea [6]. However, in most of the regions where the occultation was observable, it occurred during the daytime where the light of the Sun dominated the sky, which made the observation rather difficult to carry out. It is generally possible to observe Venus during the daytime, as the noise caused by the ambient light would be very high.

The best regions in which to observe this occultation are the western part of Java, the southern part of Sumatra and their nearby islands. In those regions, the occultation occurred just before sunrise, and the sky was not very bright. To the best of our knowledge, only observations from the Jakarta Planetarium successfully recorded the occultation process. This paper reports the results of the observations.

2. Venus circumstances

On the day of the occultation, Venus was moving away from the greatest West Elongation, which occurred on June 3, 2017. The elongation on September 18, 2017, was 27.87° West of the Sun, $50'56''$ north of the ecliptic line. Its distance from the Earth was 1.434 a.u. It shone as a -3.9 magnitude object with 87.9% of its surface facing the Earth illuminated by sunlight [7]. The calculated apparent angular diameter was $11.63''$. Venus moved fast in the sky chasing the crescent Moon, as the sky became brighter and brighter. The ephemeris of the Moon is available in [8].

The occultation started at 05:31 AM West Indonesian time, while the Sun rose at 05:21 AM in Malang city; therefore, the glare of the sunlight made Venus looked dim. However, the sky was still relatively dark in Jakarta when the occultation occurred, because the Sun rose at 05:44 AM. Both Malang and Jakarta are in a same time zone, that is West Indonesian time.

The Moon and Venus were quite close to the ecliptic (less than 1° from the ecliptic plane). Therefore, the four bodies—Sun, Earth, Moon, and Venus—were approximately in the same plane, that is, an ecliptic plane, so we present two-dimensional drawings to illustrate the situation.



Figure 1. Venus and the Moon before occultation, observed from Sunter, Jakarta.

3. Observation

The observations were carried out in three sites:

- (a) Ma Chung University, Malang, East Java ($112^{\circ} 35'22'' E$; $7^{\circ} 57'36'' S$), using a 9 cm Skywatcher BK909eq2 refractor equipped with a Nikon DSLR D7000 camera,
- (b) Sunter, North Jakarta City ($106^{\circ} 52'0'' E$; $6^{\circ} 8'2'' S$), using a Canon camera EOS 5DSR, without telescope.
- (c) Jakarta Planetarium ($106^{\circ} 50'22'' E$; $6^{\circ} 11'25'' S$), using an iOptron iE5100 digital Camera with a five-pixel-color CMOS sensor, a $1/2.5''$ chip size and a $2.2 \mu\text{m} \times 2.2 \mu\text{m}$ pixel size, which translated to a 5.76×4.29 mm detector size [9].

The observation at Ma Chung University did not yield a good result, because the occultation occurred after sunrise and the sky was quite bright. Despite the effort to remove background light by image processing, the apparent noise caused by the ambient light remained. The observation from Jakarta yielded a better result because the occultation occurred before sunrise. Although the sky was not completely dark due to scattered light, the glare of direct light from the Sun did not severely disturb Venus image, and some good images and video were obtained. A photograph of Venus few minutes before the occultation, taken from Sunter, Jakarta is presented in figure 1.

A reasonably good video recording of the process from before the occultation until the second contact resulted from the observation at Jakarta Planetarium. Further occultation phases were not observed because the sky turned too bright and no further results of good quality could be obtained. Four images were extracted from the video and combined to obtain a composite image showing the direction of movement of Venus relative to the Moon. The image is shown in figure 2.



Figure 2. The path of Venus when it was approaching the Moon, the image was extracted and prepared from video.

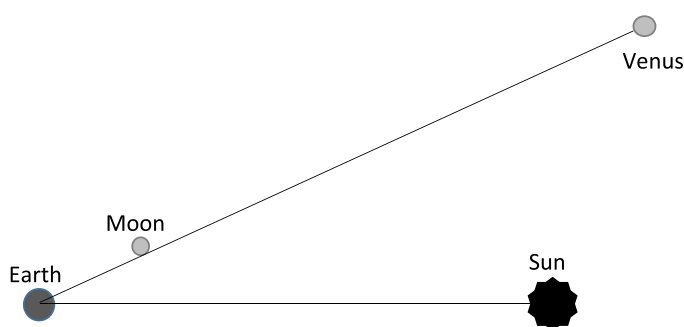


Figure 3. Geometric description of the movement of Venus toward the Moon.

4. Geometry exercise

From figure 2, we drew the traces of the path of Venus and lunar disk, and the result is illustrated in figure 3. We were able to measure the incoming angle of the path of Venus 75° relative to the disk limb at the speed of $0.843''/s$ or $50.6 \text{ arcsec}/\text{min}$. We also estimated the shortest distance between the center of the lunar disk and Venus to be 249 arcsec halfway through the occultation. From figure 1, we tried to estimate the chord of the Moon arc by manually estimating the tip of the Moon image and measuring the angle between the chord and the Venus path, which was 71° .

From the video obtained from the Jakarta Planetarium, several images near the occultation event at 14 min 1 s from the start of the video until 15 min 16 s were chosen for light curve analysis. From every four consecutive images, one image was taken to be measured and the number of frames selected was 339. The average sky photon count was calculated from a region outside Venus and the Moon to obtain the average sky photon count per pixel. Then, the

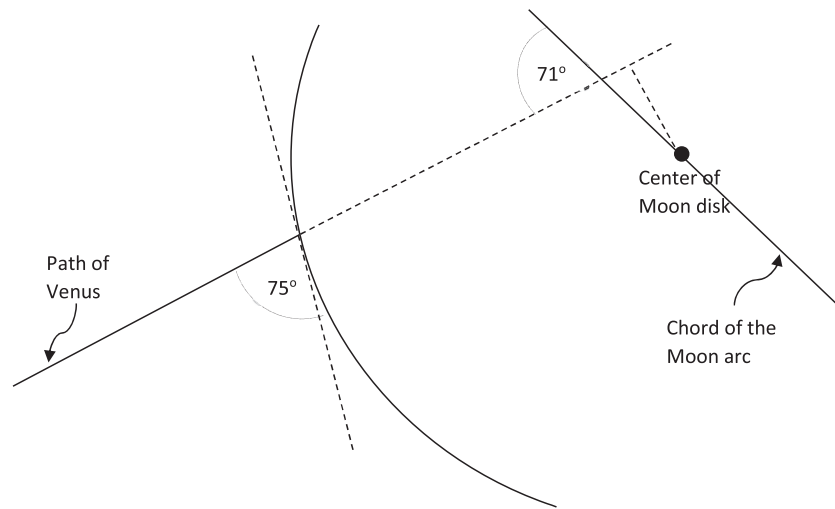


Figure 4. Geometric description of the path of Venus toward the Moon.

pixels containing Venus were subtracted from the sky count to obtain the net light count from Venus. The pixels containing Venus were selected by a criterion of a minimum count, above which they were considered to contain the Venus image. Then, the number of pixels containing Venus was counted, and their photon counts were summed for every selected frames. The integrated photon count from the pixels containing Venus were summed to obtain the Venus relative brightness. This was done for each image, and then the total count was plotted versus time, and we obtained the ingress light curve. Later we tried to fit the light curve using a simple geometric model shown in figure 5.

5. Modeling exercise

The shape of Venus on the day of occultation was gibbous. We modeled the gibbous shape as a region surrounded by two half-circles with different curvature radii. One of them was the half-circle of the Venus disk, and the other was the border of the illuminated Venus surface (see figure 5). AFBE (in figure 5) was part of the Venus disk illuminated by sunlight. The apparent angular diameter of Venus was $11.63''$, so that the radius of the first circle was $AD = BD = r_1 = 5.82''$. Let the radius of the second circle be $AC = r_2$. Then, using the Venus illumination data, 87.9% [10], r_2 can be found by solving the following equation:

$$\pi r_2^2 \left[\sin^{-1} \left(\frac{r_1}{r_2} \right) \right] - r_1 \sqrt{r_2^2 - r_1^2} = 0.379 \pi r_1^2. \quad (1)$$

The first term of the left-hand side is the area of the sector ACBE, while the second term is the area of triangle ABC. The right-hand side is the 87.9% Venus disk minus the area of half-disk AFBD. The result is $r_2 = 5.967''$. The elongation of the Moon was 28.86° . With the use of the Sun and Moon equatorial coordinates (table 1), the spherical trigonometric calculation yields the angle between Sun–Moon line and West direction to be 24.43° .

We assume that the Moon arc is a straight line because the Moon disk curvature radius is much greater than the curvature radius of the Venus disk. The error due to this assumption is not more than 0.1%. The Moon arc is presented as line GH in figure 5. It is also assumed that

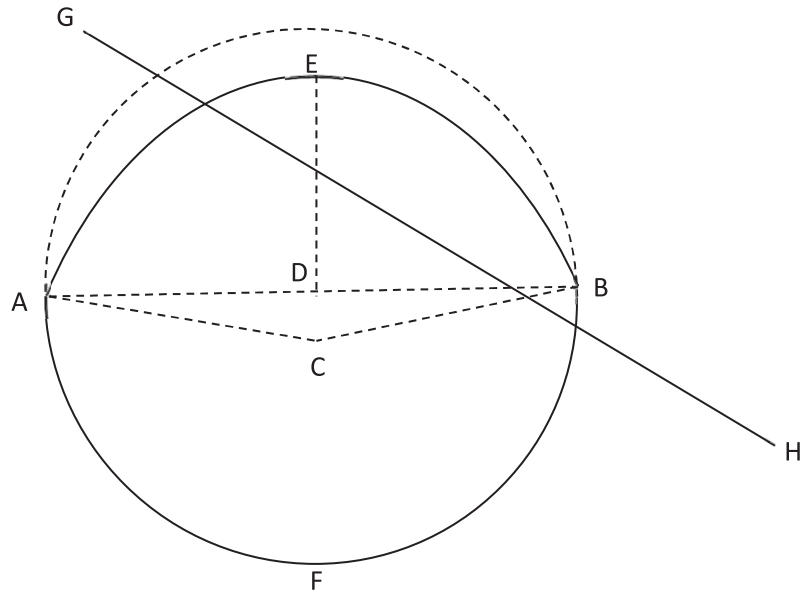


Figure 5. Geometric illustration of Venus during the occultation process. The arc AEB is the border between the dark and illuminated Venus surfaces. The arc AFB is the limb of the Venus disk. The line GH is the limb of Moon disk.

Table 1. Sun and Moon coordinate data at the beginning of occultation^a.

	RA	Dec
Sun	11 ^h 45 ^m	1° 35'
Moon	9 ^h 58 ^m	13° 6'

^aRA and Dec stands for right ascension and declination respectively.

AB is parallel to the Moon arc chord, because both objects are close and reflect light from the same source, the Sun. The angle between AB and GH was 34°. To calculate the visible area of Venus surface we calculated the area limited by the line GH, arc AEB and arc AFB.

For further calculation, we put figure 5 on the coordinate axis. We chose the AB direction as the x-axis, and then we calculated the area bordered by two circles and a straight line. As the occultation progressed, we shifted the Moon arc below and recalculated the visible area of the Venus disk. The speed of the movement of Venus was $v = 0.843''/s$ with an incoming angle of 75° relative to the straight line representing the Moon arc. With the help of figure 6, we calculated the component of speed in x and y direction.

The next step was to calculate the illuminated Venus surface that was visible by calculating the area limited by the arc AEB, straight-line GH and half-circle AFB (figure 7). This calculation was repeated with time step a quarter of a second. The component of Moon arc speed, relative to the y-axis, is $v \cos 19^\circ$; therefore, in this model, the line GH moves downward at a speed of 0.797''/s.

The first calculation was to determine when the arc AEB touches the lunar disk. The line GH is tangent to the arc AEB (figure 7). The gradient of GH is known to be $\tan(180^\circ - 34^\circ) = -0.6745$ (see figure 6), and we found $\tan \alpha = 1.4826$. Then, we found the coordinate of point L, and GD, and therefore the equation of straight-line GH was completely known.

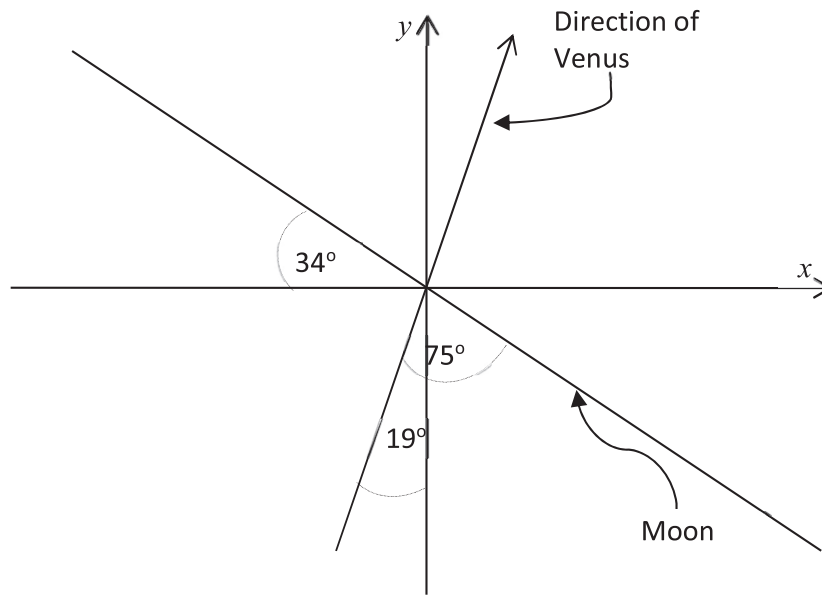


Figure 6. Illustration for calculating projected speed of Lunar disk limb relative to Venus.

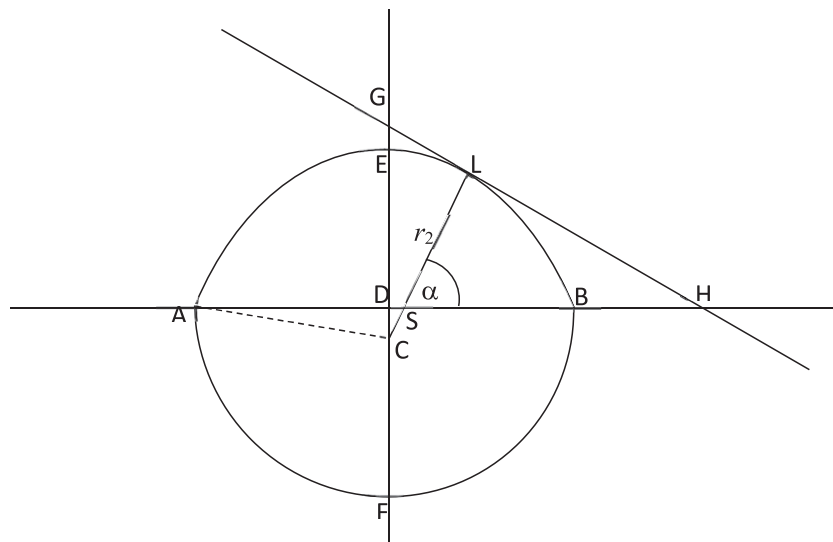


Figure 7. Illustration for calculation of unobscured Venus disk, which is the part of Venus disk below the line GH.

The next step was to move the line GH downward over a time interval 0.25 s and calculate the intersection between GH and the arc AEB. Afterward, we calculated the area of the Venus disk that was not obscured by the Moon disk, which was below the line GH. We repeated the

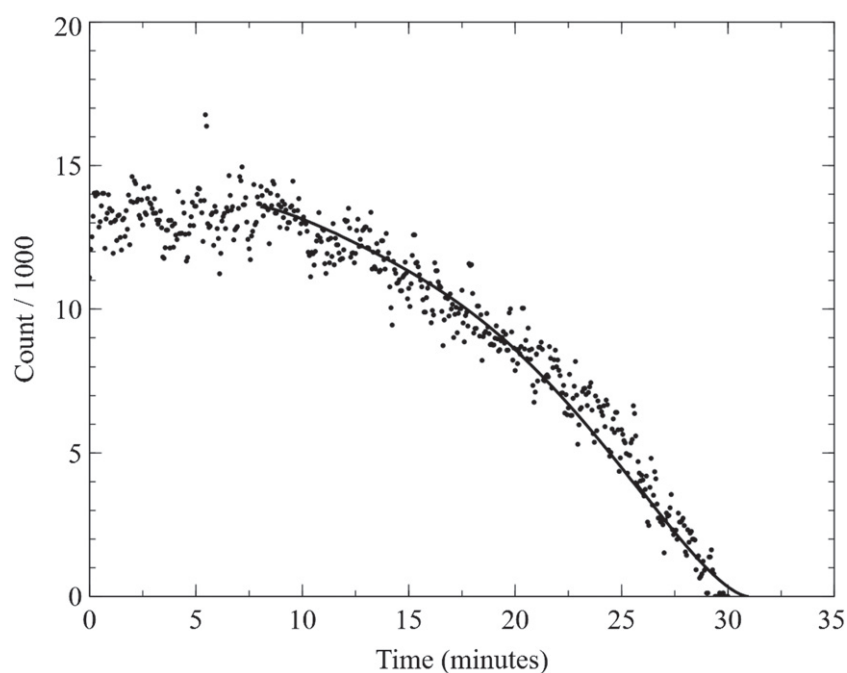


Figure 8. Fitting of the Venus light curve with the model. Dots are the count of Venus light, the line is the model.

shifting of line GH downward every 0.25 s and calculated the visible Venus area until no part of the Venus surface visible.

The result was plotted against time and fitted to the observation after count normalization. First, the model did not fit the observation. The source of the deviation was the non-uniformity of the brightness of the Venus disk. The part of the Venus surface that directly faced the Sun was the brightest, and it gradually dimmed to the limb. In our model, we simply calculated the illuminated area of Venus disk unobscured by the Moon. This calculation implies an assumption of uniform surface brightness. When we view Venus from Earth, the brightest part of the surface of Venus is not at the center of the disk but is shifted to the east. This is because the Sun is east of Venus. The occultation starts from the limb where the disk is dimmer, and the initial decrease in light flux from Venus is slow.

As the occultation progresses, the Moon increasingly blocks the brighter part, and then the decrease in Venus light becomes steeper. After the Moon blocks the brightest part of the disk the apparent dimming of Venus became shallower again. Therefore the correction factor added to the model must be increased at the beginning, reach a peak and then decrease again. The simplest mathematical function that has such behavior and can make the model fit well to the observation is in the form of $\sin^2(\phi)$. The simplest form of the correction function is preferable because this model is for education, so students will not be discouraged by complicated functions. We then introduced a correction by multiplying the area of the unobscured Venus disk by a simple correction factor $[1 + \sin^2(\phi)]$, with the $0 < \phi < 72^\circ$. The result of the fitting are presented in figure 8. The maximum angle 72° was found by trial and error. The average magnitude of relative correction is 8, 8% from the Venus total intensity. The values are near

zero in the start and the end of occultation. In between, the maximum relative correction is 16, 3%.

6. Summary and conclusion

An observation for the Venus occultation event was made in the most favorable location. The photographs and videos presented in this paper may be the only good record of the Venus occultation by the Moon on September 18, 2017. The measurement from the photograph yielded an angle between Venus path and the Moon disk of approximately 75° and the shortest distance between the center of the Moon's disk and Venus was 249 arcsec.

A simple geometric model of the occultation was constructed, and the area of the unoccluded Venus disk was calculated repeatedly as the occultation progressed. The model was fitted with observational light curve with a correction factor in the form of $1 + \sin^2(\phi)$. This correction factor is needed because the brightness of the Venus thick cloud atmosphere is not uniform, depending on the incoming angle of the sunlight. As the occultation progresses, the Moon's disk passes through brighter and brighter parts of the Venus disk. After passing the brightest part, it passes through the dimmer part. Therefore the correction function should be a function with a shape like a hill. $\sin^2(\phi)$ is one of the simplest functions that have such behavior and can make the model fit the observations.

This kind of observation and analysis is interesting to provide as a student exercise to improve their competence in geometry and computation, because it uses the observation of attractive real event. Students assessment through this kind of project is feasible during COVID-19 pandemic because students can work independently from their home using simple instruments, such as common camera, and consult with teachers online about their work.

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